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VIBRATIONS OF A FLOATING ELASTIC PLATE DUE TO PERIODIC DISPLACEMENTS OF A BOTTOM SEGMENT

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The problem of the behavior of a floating elastic thin plate under periodic vibrations of a bottom segment is solved using a numerical procedure based on the Wiener–Hopf technique. The effects of the vibration frequency, the position of the vibrating bottom segment, and the fluid depth on the vibration frequencies of the fluid and plate are studied numerically.

Key words: *surface waves, flexural-gravity waves, elastic plate, Wiener–Hopf technique, diffraction frequency.*

Introduction. Recently, the problem of the hydroelastic behavior of floating elastic plates has been studied extensively in connection with projects on the construction of floating airfields, artificial islands, and floating platforms of various applications. The huge sizes of such objects make it difficult to satisfy the similarity criteria in experimental studies; therefore, numerical modeling plays a great role in their analysis.

The diffraction of surface waves on a floating elastic plate has been studied fairly well. Considerable less attention has been paid to the forced vibrations of a plate subjected to unsteady loading and the behavior of a floating elastic plate under earthquake-induced vibrations of a bottom segment. High-frequency vibrations are studied in [1], where the bottom is modeled by a homogeneous elastic medium (half-space), in which compression and shear waves propagate from the earthquake epicenter and the fluid is considered compressible and imponderable. An analytical solution of the problem of an elastic semi-infinite plate for specified periodic vibrations of the bottom and an incompressible imponderable fluid was obtained in [2] using the Wiener–Hopf technique. A review of existing numerical techniques for studying the behavior of floating elastic plates is given in [3]. In the present paper, the Wiener–Hopf technique is employed to study the vibrations of a plate of finite width floating on the surface of an incompressible ponderable fluid of finite depth under vibrations of a bottom segment in a plane formulation. The effects of the vibration frequency, the position of the vibrating segment, the and fluid depth on the vibration amplitudes of the fluid and plate are investigated for conditions of a model airport.

1. Formulation. The hydroelastic behavior of a floating plate under periodic vibrations of a bottom segment is studied using linear theory. The plate has thickness h and length L_0. The left edge of the plate is taken to be the origin of Cartesian coordinates Oxy. The plate edges are not fixed. The fluid is ideal and incompressible, and its flow is vortex-free. The plate thickness is assumed to be much smaller than the length of the waves propagating in the plate. We use the model of thin plates. The boundary conditions are extended to the unperturbed water surface.

The fluid-velocity potential φ satisfies the Laplace equation and the boundary conditions

$$\Delta\varphi = 0 \qquad (-H_0 < y < 0),$$

$$\varphi_y = w_t \quad (y = 0), \qquad w(x, -H_0, t) = u(x)\,\mathrm{e}^{-i\omega t},$$

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$$D\,\frac{\partial^4 w}{\partial x^4} + \rho_0 h\,\frac{\partial^2 w}{\partial t^2} = p \qquad (y = 0, \quad 0 < x < L_0), \tag{1.1}$$

$$p = -\rho(\varphi_t + gw),$$

$$\varphi_t + gw = 0 \qquad (y = 0, \quad x \in (-\infty, 0) \cup (L_0, \infty)).$$

Here H_0 is the fluid depth, w is the vertical displacement of the upper surface of the fluid (plate), p is the hydrodynamic pressure, g is the acceleration of gravity, D is the flexural rigidity of the plate, ρ and ρ_0 are the densities of the fluid and the plate, t is time, and ω is the vibration frequency. At the edges of the plate, the moment and shear force should vanish:

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^3 w}{\partial x^3} = 0 \qquad (y = 0, \quad x = 0, L_0). \tag{1.2}$$

We first consider the case of a point source at the bottom: $u(x,t) = u_0\delta(x - x_0)$. The time dependence of all functions is expressed by the factor $\mathrm{e}^{-i\omega t}$. We introduce the characteristic length $l = g/\omega^2$ and the dimensionless variables

$$x' = \frac{x}{l}, \quad y' = \frac{y}{l}, \quad \varphi' = \frac{\omega\varphi}{gu_0}, \quad w' = \frac{w}{u_0}, \quad t' = \omega t$$

(below, the primes are omitted). The potential is written as $\varphi = \phi(x,y)\,\mathrm{e}^{-it}$. Then, from (1.1) and (1.2), we obtain the following boundary-value problem for ϕ:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (-H < y < 0);$$

$$\frac{\partial \phi}{\partial y} = -i\delta(x - x_*) \qquad (y = -H); \tag{1.3}$$

$$\frac{\partial \phi}{\partial y} - \phi = 0 \qquad (y = 0, \quad x \in (-\infty, 0) \cup (L, \infty)); \tag{1.4}$$

$$\Big(\beta\,\frac{\partial^4}{\partial x^4} + 1 - d\Big)\frac{\partial \phi}{\partial y} - \phi = 0 \qquad (y = 0, \quad 0 < x < L); \tag{1.5}$$

$$\frac{\partial^2}{\partial x^2}\,\frac{\partial \phi}{\partial y} = \frac{\partial^3}{\partial x^3}\,\frac{\partial \phi}{\partial y} = 0 \qquad (y = 0, \quad x = 0, L); \tag{1.6}$$

$$L = \frac{L_0}{l}, \quad H = \frac{H_0}{l}, \quad x_* = \frac{x_0}{l}, \quad \beta = \frac{D}{\rho g l^4}, \quad d = \frac{\rho_0 h}{\rho l}.$$

Here L, H, x_*, β, and d are the dimensionless parameters of the problem (the plate length, the fluid depth, the position of the source of vibrations, the reduced rigidity, and the draft of the plate, respectively). In addition, the radiation conditions for $|x| \to \infty$ and the regularity conditions at the edges (the local boundedness of energy) should be satisfied.

2. System of Integral Equations. The problem is solved using the Wiener–Hopf technique in the Jones interpretation [4]. Let us consider the following functions of the complex variable α:

$$\Phi_+(\alpha, y) = \int_L^\infty \mathrm{e}^{i\alpha(x-L)}\,\phi(x,y)\,dx, \qquad \Phi_-(\alpha, y) = \int_{-\infty}^0 \mathrm{e}^{i\alpha x}\,\phi(x,y)\,dx,$$

$$\Phi_1(\alpha, y) = \int_0^L \mathrm{e}^{i\alpha x}\,\phi(x,y)\,dx, \qquad \Phi(\alpha, y) = \Phi_-(\alpha, y) + \Phi_1(\alpha, y) + \mathrm{e}^{i\alpha L}\,\Phi_+(\alpha, y). \tag{2.1}$$

The function $\Phi_+(\alpha, y)$ is defined in the upper semiplane $\operatorname{Im}\alpha > 0$, and $\Phi_-(\alpha, y)$ in the lower semiplane $\operatorname{Im}\alpha < 0$. These functions can be defined on the entire complex plane by analytic continuation.

For surface waves, the values of α should satisfy the dispersion equation

$$K_1(\alpha) \equiv \alpha \tanh\,(\alpha H) - 1 = 0,$$

which has two real roots $\pm\gamma$ and a countable set of purely imaginary roots $\pm\gamma_n$ $(n = 1, 2, \ldots)$ symmetric about the real axis [4]; $\gamma_n \to in\pi/H$ as $n \to \infty$.

For the flexural-gravity waves propagating in the plate, we obtain the dispersion relation

$$K_2(\alpha) \equiv (\beta\alpha^4 + 1 - d)\alpha \tanh\,(\alpha H) - 1 = 0,$$

which has two real roots $\pm\alpha_0$, a countable set purely imaginary roots $\pm\alpha_n$ $(n = 1, 2, \ldots)$ symmetric about the real axis, and four complex roots symmetric about the real and imaginary axes [5]. We denote the root lying in the first quadrant by α_{-1} and the root in the second quadrant by α_{-2}; $\alpha_n \to in\pi/H$ as $n \to \infty$.

The dispersion functions $K_1(\alpha)$ and $K_2(\alpha)$ are even. The real roots of the dispersion relations define propagating waves, and the remaining roots define edge waves, which damp exponentially away from the perturbation source.

We study the behavior of the functions $\Phi_\pm(\alpha, y)$. For $x \to -\infty$, the potential represents a wave of the form $R\,\mathrm{e}^{-i\gamma x}$ (R is the complex amplitude of the wave propagating to the left) and a set of exponentially damped waves. The least damped wave corresponds to the root γ_1. Then, $\Phi_-(\alpha, y)$ is analytic in the semiplane $\operatorname{Im}\alpha < |\gamma_1|$ except for the pole at $\alpha = \gamma$. For $x \to \infty$, the potential ϕ represents a wave of the form $T\,\mathrm{e}^{i\gamma x}$ (T is the complex amplitude of the wave propagating to the right) and a set of exponentially damped modes. Therefore, the function $\Phi_+(\alpha, y)$ is analytic in the semiplane $\{\operatorname{Im}\alpha > -|\gamma_1|\}$ except for the pole at the point $\alpha = -\gamma$.

The function $\Phi(\alpha, y)$ is the Fourier transform of the function $\phi(x, y)$ and satisfies the equation

$$\frac{\partial^2\Phi}{\partial y^2} - \alpha^2\Phi = 0.$$

The general solution of this equation has the form

$$\Phi(\alpha, y) = C(\alpha)Z(\alpha, y) + S(\alpha)\sinh\,(\alpha(y + H)), \qquad Z(\alpha, y) = \cosh\,(\alpha(y + H))/\cosh\,(\alpha H). \tag{2.2}$$

From condition (1.3) at the bottom, we obtain

$$\frac{\partial\Phi}{\partial y}(\alpha, -H) = -i\,\mathrm{e}^{i\alpha x_*}, \qquad S(\alpha) = -\frac{i\,\mathrm{e}^{i\alpha x_*}}{\alpha}.$$

We denote by $D_\pm(\alpha)$ and $D_1(\alpha)$ integrals of the form (2.1) in which the integrand ϕ is replaced by the left side of the boundary condition (1.4), and by $F_\pm(\alpha)$ and $F_1(\alpha)$ similar expressions in which the integrand is the left side of expression (1.5). Let us consider the functions

$$D(\alpha) = D_-(\alpha) + D_1(\alpha) + \mathrm{e}^{i\alpha L}\,D_+(\alpha), \qquad F(\alpha) = F_-(\alpha) + F_1(\alpha) + \mathrm{e}^{i\alpha L}\,F_+(\alpha).$$

The functions $D(\alpha)$ and $F(\alpha)$ are the Fourier transforms of the dispersion functions, which will be understood in the sense of generalized functions [6]. They satisfy the relations

$$D(\alpha) = \frac{\partial\Phi}{\partial y}(\alpha, 0) - \Phi(\alpha, 0), \qquad F(\alpha) = (\beta\alpha^4 + 1 - d)\,\frac{\partial\Phi}{\partial y}(\alpha, 0) - \Phi(\alpha, 0).$$

From boundary conditions (1.4) and (1.5), we have

$$D_-(\alpha) = D_+(\alpha) = 0, \qquad F_1(\alpha) = 0,$$

$$D_1(\alpha) = D(\alpha) = C(\alpha)K_1(\alpha) - i\,\mathrm{e}^{i\alpha x_*}\Big(\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha\Big), \tag{2.3}$$

$$F_-(\alpha) + \mathrm{e}^{i\alpha L}\,F_+(\alpha) = C(\alpha)K_2(\alpha) - i\,\mathrm{e}^{i\alpha x_*}\Big[(\beta\alpha^4 + 1 - d)\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha\Big].$$

Eliminating $C(\alpha)$ from these relations, we obtain the equation

$$F_-(\alpha) + \mathrm{e}^{i\alpha L}\,F_+(\alpha) + i\,\mathrm{e}^{i\alpha x_*}\Big[(\beta\alpha^4 + 1 - d)\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha\Big]$$
$$= K(\alpha)\Big[D_1(\alpha) + i\,\mathrm{e}^{i\alpha x_*}\Big(\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha\Big)\Big], \tag{2.4}$$

$$K(\alpha) = K_2(\alpha)/K_1(\alpha).$$

According to the Wiener–Hopf technique, it is necessary to factorize the function $K(\alpha)$, i.e., to write it as

$$K(\alpha) = K_+(\alpha)K_-(\alpha),$$

where the functions $K_\pm(\alpha)$ are regular in the same regions as the function $\Phi_\pm(\alpha, y)$. The function $K(\alpha)$ has zeroes and poles at the points $\pm\gamma$ and $\pm\alpha_0$, respectively, on the real axis. We therefore consider the analyticity regions Π_+ and Π_-, where Π_+ is the semiplane $\operatorname{Im}\alpha > -|\gamma_1|$ with cuts eliminating the points $-\alpha_0$ and $-\gamma$ and Π_- is the semiplane $\operatorname{Im}\alpha < |\gamma_1|$ with cuts eliminating the points α_0 and γ.

Let us introduce the function

$$g(\alpha) = \frac{K(\alpha)(\alpha^2 - \gamma^2)}{\beta(\alpha^2 - \alpha_0^2)(\alpha^2 - \alpha_{-1}^2)(\alpha^2 - \alpha_{-2}^2)}.$$

The function $g(\alpha)$ does not have zeroes on the real axis, is bounded, and tends to unity at infinity. It is factorized as follows [4]:

$$g(\alpha) = g_+(\alpha)g_-(\alpha), \qquad g_\pm(\alpha) = \exp\Big[\pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{\ln g(x)}{x-\alpha}\,dx\Big], \quad \sigma < |\gamma_1|.$$

The functions $K_\pm(\alpha)$ are defined by the formula

$$K_\pm(\alpha) = \frac{\sqrt{\beta}(\alpha\pm\alpha_0)(\alpha\pm\alpha_{-1})(\alpha\pm\alpha_{-2})g_\pm(\alpha)}{\alpha\pm\gamma}.$$

In this case, $K_+(\alpha) = K_-(-\alpha)$. We multiply Eq. (2.4) by $\mathrm{e}^{-i\alpha L}[K_+(\alpha)]^{-1}$ and bring it to the form

$$\frac{F_+(\alpha)}{K_+(\alpha)} + \frac{\mathrm{e}^{-i\alpha L}F_-(\alpha)}{K_+(\alpha)} - \frac{i\,\mathrm{e}^{i\alpha(x_*-L)}(\beta\alpha^4-d)}{\cosh(\alpha H)K_+(\alpha)K_1(\alpha)} = D_1(\alpha)K_-(\alpha)\,\mathrm{e}^{-i\alpha L}.$$

Representing the terms on the left side of this equation as the decomposition

$$\frac{\mathrm{e}^{-i\alpha L}F_-(\alpha)}{K_+(\alpha)} = U_+(\alpha)+U_-(\alpha), \qquad -\frac{i\,\mathrm{e}^{i\alpha(x_*-L)}(\beta\alpha^4-d)}{\cosh(\alpha H)K_+(\alpha)K_1(\alpha)} = L_+(\alpha)+L_-(\alpha),$$

we write

$$F_+(\alpha)/K_+(\alpha) + U_+(\alpha) + L_+(\alpha) = D_1(\alpha)K_-(\alpha)\,\mathrm{e}^{-i\alpha L} - L_-(\alpha) - U_-(\alpha). \tag{2.5}$$

We now divide Eq. (2.4) by $K_-(\alpha)$ and bring it to the form

$$\frac{F_-(\alpha)}{K_-(\alpha)} + \frac{\mathrm{e}^{i\alpha L}F_+(\alpha)}{K_-(\alpha)} - \frac{i\,\mathrm{e}^{i\alpha x_*}(\beta\alpha^4-d)}{\cosh(\alpha H)K_-(\alpha)K_1(\alpha)} = D_1(\alpha)K_+(\alpha).$$

Representing the terms on the left side of this equation as the decomposition

$$\frac{\mathrm{e}^{i\alpha L}F_+(\alpha)}{K_-(\alpha)} = V_+(\alpha)+V_-(\alpha), \qquad -\frac{i\,\mathrm{e}^{i\alpha x_*}(\beta\alpha^4-d)}{\cosh(\alpha H)K_-(\alpha)K_1(\alpha)} = N_+(\alpha)+N_-(\alpha),$$

we write

$$F_-(\alpha)/K_-(\alpha) + V_-(\alpha) + N_-(\alpha) = D_1(\alpha)K_+(\alpha) - V_+(\alpha) - N_+(\alpha). \tag{2.6}$$

The functions $U_\pm(\alpha)$, $V_\pm(\alpha)$, $L_\pm(\alpha)$, and $N_\pm(\alpha)$ are defined by the expressions [4]

$$U_\pm(\alpha) = \pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{\mathrm{e}^{-i\zeta L}F_-(\zeta)\,d\zeta}{K_+(\zeta)(\zeta-\alpha)}; \qquad V_\pm(\alpha) = \pm\frac{1}{2\pi i}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{\mathrm{e}^{i\zeta L}F_+(\zeta)\,d\zeta}{K_-(\zeta)(\zeta-\alpha)};$$

$$L_\pm(\alpha) = \mp\frac{1}{2\pi}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{\mathrm{e}^{i\zeta(x_*-L)}(\beta\zeta^4-d)\,d\zeta}{\cosh(\alpha H)K_+(\zeta)K_1(\zeta)(\zeta-\alpha)}; \tag{2.7}$$

$$N_\pm(\alpha) = \mp\frac{1}{2\pi}\int\limits_{-\infty\mp i\sigma}^{\infty\mp i\sigma}\frac{\mathrm{e}^{i\zeta x_*}(\beta\zeta^4-d)\,d\zeta}{\cosh(\alpha H)K_-(\zeta)K_1(\zeta)(\zeta-\alpha)}, \tag{2.8}$$

where $\sigma < |\gamma_1|$.

The left side of Eq. (2.5) contains a function analytic in the region Π_+ and the right side contains a function analytic in the region Π_-. Analytic continuation yields a function analytic in the entire complex plane. According to Liouville's theorem, this function is a polynomial. The degree of the polynomial is determined by the behavior of the functions as $|\alpha| \to \infty$. The condition of local boundedness of energy implies that at the plate edge, the velocities have a singularity of order not higher than $O(r^{-\lambda})$ ($\lambda < 1$ and r is the distance to the plate edge). Then, for $|\alpha| \to \infty$, the function $F_-(\alpha)$ has order not higher than $O(|\alpha|^{\lambda+3})$ and $D_+(\alpha)$ has order not higher than $O(|\alpha|^{\lambda-1})$ [6]. At infinity, the functions $K_\pm(\alpha)$ have order $O(|\alpha|^2)$ because $g_\pm(\alpha) \to 1$ as $|\alpha| \to \infty$. Therefore, the degree of the polynomial is equal to unity and

$$F_+(\alpha)/K_+(\alpha) + U_+(\alpha) + L_+(\alpha) = a_1 + a_2\alpha. \tag{2.9}$$

Similarly, from Eq. (2.6) we have

$$F_-(\alpha)/K_-(\alpha) + V_-(\alpha) + N_-(\alpha) = b_1 + b_2\alpha. \tag{2.10}$$

Here a_1, a_2, b_1, and b_2 are unknown constants determined from the edge conditions (1.6).

Equations (2.9) and (2.10) yield the system

$$\begin{aligned} &\frac{F_+(\alpha)}{K_+(\alpha)} + \frac{1}{2\pi i}\int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{e^{-i\zeta L}\, F_-(\zeta)\, d\zeta}{(\zeta-\alpha)K_+(\zeta)} = a_1 + a_2\alpha - L_+(\alpha), \\ &\frac{F_-(\alpha)}{K_-(\alpha)} - \frac{1}{2\pi i}\int\limits_{-\infty-i\sigma}^{\infty-i\sigma} \frac{e^{i\zeta L}\, F_+(\zeta)\, d\zeta}{(\zeta-\alpha)K_-(\zeta)} = b_1 + b_2\alpha - N_-(\alpha). \end{aligned} \tag{2.11}$$

Let us determine the constants a_1 and a_2. We have

$$D_1(\alpha)K_-(\alpha)\, e^{-i\alpha L} - L_-(\alpha) - U_-(\alpha) = a_1 + a_2\alpha.$$

Substitution of the expression for $U_-(\alpha)$ into this equation yields

$$D_1(\alpha) = \frac{e^{i\alpha L}}{K_-(\alpha)}\Big[a_1 + a_2\alpha + L_-(\alpha) - \frac{1}{2\pi i}\int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{e^{-i\zeta L}\, F_-(\zeta)\, d\zeta}{K_+(\zeta)(\zeta-\alpha)}\Big].$$

In view of (2.2) and (2.3), the inverse Fourier transformation yields the following expression for the potential:

$$\begin{aligned} \phi(x,y) = {} & \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha(x-L)}\cosh\,(\alpha(y+H))}{\cosh\,(\alpha H)K_-(\alpha)K_1(\alpha)}\Big[a_1 + a_2\alpha + L_-(\alpha) - \frac{1}{2\pi i}\int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{e^{-i\zeta L}\, F_-(\zeta)\, d\zeta}{K_+(\zeta)(\zeta-\alpha)}\Big]\, d\alpha \\ & + \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha(x-x_*)}\sinh\,(\alpha(y+H))\, d\alpha}{\alpha} \\ & - \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha(x-x_*)}(\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha)Z(\alpha,y)\, d\alpha}{K_1(\alpha)}. \end{aligned} \tag{2.12}$$

Multiplying the numerator and denominator by $K_+(\alpha)$ and performing some transformations, we obtain the following expression for the derivative of the potential:

$$\begin{aligned} \frac{\partial\phi}{\partial y}(x,0) = {} & \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha(x-L)}\,\alpha\tanh\,(\alpha H)K_+(\alpha)}{K_2(\alpha)}\Big[a_1 + a_2\alpha - L_+(\alpha) - \frac{1}{2\pi i}\int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{e^{-i\zeta L}\, F_-(\zeta)\, d\zeta}{K_+(\zeta)(\zeta-\alpha)}\Big]\, d\alpha \\ & - \frac{1}{2\pi i}\int\limits_{-\infty}^{\infty} \frac{e^{-i\alpha(x-x_*)}\, d\alpha}{\cosh\,(\alpha H)K_2(\alpha)}. \end{aligned} \tag{2.13}$$

The integration contour for the outer integral should be chosen such that it completely lies in the intersection of the regions Π_+ and Π_-. It is possible to choose the integration contour on the real axis so that encircles the points α_0 and γ from below and the points $-\alpha_0$ and $-\gamma$ from above.

In the inner integral, $\operatorname{Im}\alpha < \sigma$. However, this integral as a function of α can be defined by analytic continuation on the entire complex plane. This integral is calculated using residue theory. The function $K_+(\zeta)$ has zeroes at the points $-\alpha_j$ $(j = -2, -1, 0, \ldots)$ and poles at the points $-\gamma, -\gamma_j$ $(j = 1, 2, \ldots)$. The integrand has poles at the points $\zeta = -\alpha_j$ $(j = -2, -1, 0, \ldots)$ and at the point $\zeta = \alpha$. Therefore,

$$\frac{1}{2\pi i}\int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{\mathrm{e}^{-i\zeta L} F_-(\zeta)\, d\zeta}{K_+(\zeta)(\zeta-\alpha)} = -\frac{\mathrm{e}^{-i\alpha L} F_-(\alpha)}{K_+(\alpha)} + \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j L} F_-(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha)}.$$

We consider the case $x_* < x < L$. The outer integral in (2.13) is also calculated using residue theory. In the first and third integrals, the integration contour for α is closed in the upper semiplane, and in the second integral, it is closed in the lower semiplane. We obtain

$$\frac{\partial\phi}{\partial y}(x,0) = i\sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m(L-x)}\, \alpha_m \tanh\,(\alpha_m H) K_+(\alpha_m)}{K_2'(\alpha_m)}$$

$$\times\Big[a_1 + a_2\alpha_m - L_+(\alpha_m) - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j L} F_-(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)}\Big]$$

$$-\, i\sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m x}\, \alpha_m \tanh\,(\alpha_m H) F_-(-\alpha_m)}{K_2'(-\alpha_m)} - \sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m|x-x_*|}}{\cosh\,(\alpha_m H) K_2'(\alpha_m)}.$$

Substitution of the above expression into boundary conditions (1.6) for $x = L$ yields two equations:

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^n \tanh\,(\alpha_m H) K_+(\alpha_m)}{K_2'(\alpha_m)}\Big[a_1 + a_2\alpha_m - L_+(\alpha_m) - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j L} F_-(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)}\Big]$$

$$+\,(-1)^n \sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m L}\, \alpha_m^n \tanh\,(\alpha_m H) F_-(-\alpha_m)}{K_2'(-\alpha_m)} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\alpha_m^{n-1}\, \mathrm{e}^{i\alpha_m(L-x_*)}\, d\alpha}{\cosh\,(\alpha_m H) K_2'(\alpha_m)} = 0,$$

$$n = 3, 4. \tag{2.14}$$

The dispersion relation under the plate implies

$$\alpha_m \tanh\,(\alpha_m H) = -K_1(\alpha_m)/(\beta\alpha_m^4 - d).$$

Substitution of this expression into (2.14) yields

$$\sum_{m=-2}^{\infty} \frac{\alpha_m^{n-1} K_+(\alpha_m) K_1(\alpha_m)}{K_2'(\alpha_m)(\beta\alpha_m^4 - d)}\Big[a_1 + a_2\alpha_m - L_+(\alpha_m) - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j L} F_-(-\alpha_j)}{K_+'(-\alpha_j)(\alpha_j+\alpha_m)}\Big]$$

$$+\,(-1)^n \sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m L}\, \alpha_m^{n-1} F_-(-\alpha_m) K_1(-\alpha_m)}{K_2'(-\alpha_m)(\beta\alpha_m^4 - d)} + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\alpha_m^{n-1}\, \mathrm{e}^{i\alpha_m(L-x_*)}\, d\alpha}{\cosh\,(\alpha_m H) K_2'(\alpha_m)} = 0,$$

$$n = 3, 4. \tag{2.15}$$

We note that the first term in (2.15) is the sum of residues at the points α_m for the integral

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\alpha^{n-1} K_1(\alpha) K_+(\alpha)[a_1 + a_2\alpha - L_+(\alpha)]\, d\alpha}{K_2(\alpha)(\beta\alpha^4 - d)} = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{\alpha^{n-1}[a_1 + a_2\alpha - L_+(\alpha)]\, d\alpha}{K_-(\alpha)(\beta\alpha^4 - d)}.$$

We reduce this integral to the sum of residues in the roots of the polynomial $\beta\alpha^4 - d$. Proceeding similarly with the remaining sums, we obtain the following system of equations for the constants a_1 and a_2:

$$a_1 \sum_{k=1}^{4} z_k^{n-2} K_+(z_k) + a_2 \sum_{k=1}^{4} z_k^{n-1} K_+(z_k)$$

$$= \sum_{k=1}^{4} z_k^{n-2} K_+(z_k) \Big[L_+(z_k) - \frac{1}{2\pi i} \int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{\mathrm{e}^{-i\zeta L}\, F_-(\zeta)\, d\zeta}{K_+(\zeta)(z_k-\zeta)} \Big], \quad n = 1, 2, \tag{2.16}$$

where z_k are roots of the polynomial $\beta\alpha^4 - d$.

Let us now determine the constants b_1 and b_2. From Eqs. (2.6) and (2.10), we have

$$D_1(\alpha)K_+(\alpha) - V_+(\alpha) - N_+(\alpha) = b_1 + b_2\alpha.$$

Substitution of the expression for $V_+(\alpha)$ and $N_+(\alpha)$ into this equation yields

$$D_1(\alpha) = \frac{1}{K_+(\alpha)} \Big[b_1 + b_2\alpha + N_+(\alpha) + \frac{1}{2\pi i} \int\limits_{-\infty-i\sigma}^{\infty-i\sigma} \frac{\mathrm{e}^{i\zeta L}\, F_+(\zeta)\, d\zeta}{K_-(\zeta)(\zeta-\alpha)} \Big].$$

The inverse Fourier transformation with allowance for (2.2) and (2.3) leads to the following representation for the potential ϕ:

$$\phi(x,y) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha x}\, Z(\alpha,y)}{K_+(\alpha)K_1(\alpha)} \Big[b_1 + b_2\alpha + N_+(\alpha) + \frac{1}{2\pi i} \int\limits_{-\infty-i\sigma}^{\infty-i\sigma} \frac{\mathrm{e}^{i\zeta L}\, F_+(\zeta)\, d\zeta}{K_-(\zeta)(\zeta-\alpha)} \Big] d\alpha$$

$$-\frac{1}{2\pi i} \int\limits_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)}\, Z(\alpha,y)(\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha) d\alpha}{K_1(\alpha)}$$

$$+ \frac{1}{2\pi i} \int\limits_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)} \sinh\,(\alpha(y+H))\, d\alpha}{\alpha}. \tag{2.17}$$

For the derivative of the potential, we obtain the expression

$$\frac{\partial\phi}{\partial y}(x,0) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha x}\, \alpha \tanh\,(\alpha H) K_-(\alpha)}{K_2(\alpha)} \Big[b_1 + b_2\alpha + N_+(\alpha) + \frac{1}{2\pi i} \int\limits_{-\infty-i\sigma}^{\infty-i\sigma} \frac{\mathrm{e}^{i\zeta L}\, F_+(\zeta) d\zeta}{K_-(\zeta)(\zeta-\alpha)} \Big] d\alpha$$

$$- \frac{1}{2\pi i} \int\limits_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)}\, d\alpha}{\cosh\,(\alpha H) K_2(\alpha)}.$$

Substitution of this expression into boundary conditions (1.6) leads to the following system of equations for b_1 and b_2:

$$b_1 \sum_{k=1}^{4} \frac{z_k^{n-2}}{K_+(z_k)} + b_2 \sum_{k=1}^{4} \frac{z_k^{n-1}}{K_+(z_k)} = \sum_{k=1}^{4} \frac{z_k^{n-2}}{K_+(z_k)} \Big[-N_+(z_k) + \frac{1}{2\pi i} \int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{\mathrm{e}^{-i\zeta L}\, F_-(\zeta)\, d\zeta}{K_+(\zeta)(z_k-\zeta)} \Big], \qquad n = 1, 2. \tag{2.18}$$

Thus, the problem reduces to solving system (2.11), (2.16), (2.18).

3. Solution of the System. The integrals in Eqs. (2.11), (2.16), and (2.18) are calculated using residue theory. We introduce the new unknowns

$$\xi_j = \frac{F_+(\alpha_j)}{\alpha_j^2 K_+(\alpha_j)}, \qquad \eta_j = \frac{F_-(-\alpha_j)}{\alpha_j^2 K_-(-\alpha_j)}$$

for which we obtain the system

$$
\begin{gathered}
\boldsymbol{\xi} - C\boldsymbol{\eta} - A\boldsymbol{a} = \boldsymbol{F}_1, \\
\boldsymbol{\eta} - C\boldsymbol{\xi} - \tilde{A}\boldsymbol{b} = \boldsymbol{F}_2, \\
G\boldsymbol{\eta} + B\boldsymbol{a} = \boldsymbol{F}_3, \\
S\boldsymbol{\xi} + D\boldsymbol{b} = \boldsymbol{F}_4.
\end{gathered} \tag{3.1}
$$

Here $\boldsymbol{\xi}$, $\boldsymbol{\eta}$, $\boldsymbol{F}_n$, $\boldsymbol{a}$, and $\boldsymbol{b}$ are the vectors $\{\xi_j\}$, $\{\eta_j\}$, $\{F_j^{(n)}\}$, $\{a_i\}$, and $\{b_i\}$ and C, G, S, B, D, A, and $\tilde{A}$ are the matrices $\{C_{jm}\}$, $\{G_{im}\}$, $\{S_{im}\}$, $\{B_{im}\}$, $\{D_{im}\}$, $\{A_{ji}\}$, and $\{\tilde{A}_{ji}\}$:

$$
C_{jm} = \frac{Q_m}{\alpha_j^2(\alpha_m + \alpha_j)}; \qquad Q_m = \frac{\mathrm{e}^{i\alpha_m L}\,\alpha_m^2 K_+^2(\alpha_m) K_1(\alpha_m)}{K_2'(\alpha_m)}; \qquad \tilde{A}_{ji} = (-1)^{i-1}\alpha_j^{i-3};
$$

$$
A_{ji} = \alpha_j^{i-3}; \qquad G_{im} = Q_m \sum_{k=1}^{4} \frac{z_k^{i-2} K_+(z_k)}{z_k + \alpha_j}; \qquad S_{im} = -Q_m \sum_{k=1}^{4} \frac{z_k^{i-2}}{K_+(z_k)(z_k - \alpha_j)};
$$

$$
B_{11} = \sum_{k=1}^{4} \frac{K_+(z_k)}{z_k}, \qquad B_{12} = B_{21} = \sum_{k=1}^{4} K_+(z_k); \qquad B_{22} = \sum_{k=1}^{4} z_k K_+(z_k);
$$

$$
D_{11} = \sum_{k=1}^{4} \frac{1}{K_+(z_k) z_k}; \qquad D_{12} = D_{21} = \sum_{k=1}^{4} \frac{1}{K_+(z_k)}; \qquad D_{22} = \sum_{k=1}^{4} \frac{z_k}{K_+(z_k)};
$$

$$
F_j^{(1)} = -L_+(\alpha_j)/\alpha_j^2; \qquad F_j^{(2)} = -N_-(-\alpha_j)/\alpha_j^2;
$$

$$
F_i^{(3)} = \sum_{k=1}^{4} z_k^{i-2} K_+(z_k) L_+(z_k); \qquad F_i^{(4)} = \sum_{k=1}^{4} z_k^{i-2} N_-(z_k)/K_+(z_k);
$$

$$
L_+(\alpha) = \begin{cases} -i \displaystyle\sum_{m=0}^{\infty} \frac{\mathrm{e}^{i\gamma_m(x_*-L)}(\beta\gamma_m^4 - d)}{\cosh\,(\gamma_m H) K_+(\gamma_m) K_1'(\gamma_m)(\gamma_m - \alpha)} - i\,\frac{\mathrm{e}^{i\alpha(x_*-L)}(\beta\alpha^4 - d)}{\cosh\,(\alpha H) K_+(\alpha) K_1(\alpha)}, & x_* > L, \\ i \displaystyle\sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m(L-x_*)}\,K_+(\alpha_m)(\beta\alpha_m^4 - d)}{\cosh\,(\alpha_m H) K_2'(\alpha_m)(\alpha_m + \alpha)}, & x_* < L; \end{cases}
$$

$$
N_-(\alpha) = \begin{cases} -i \displaystyle\sum_{m=0}^{\infty} \frac{\mathrm{e}^{-i\gamma_m x_*}(\beta\gamma_m^4 - d)}{\cosh\,(\gamma_m H) K_+(\gamma_m) K_1'(\gamma_m)(\gamma_m + \alpha)} - i\,\frac{\mathrm{e}^{i\alpha x_*}(\beta\alpha^4 - d)}{\cosh\,(\alpha H) K_-(\alpha) K_1(\alpha)}, & x_* < 0, \\ i \displaystyle\sum_{m=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_m x_*}\,K_+(\alpha_m)(\beta\alpha_m^4 - d)}{\cosh\,(\alpha_m H) K_2'(\alpha_m)(\alpha_m - \alpha)}, & x_* > 0. \end{cases}
$$

After solving system (3.1), we find the deflection of the plate and the elevation of the free surface away from the plate. We have

$$
C(\alpha) = \Big(F_-(\alpha) + \mathrm{e}^{i\alpha L}\,F_+(\alpha) + i\,\mathrm{e}^{i\alpha_0 x_*}[(\beta\alpha^4 + 1 - d)\cosh\,(\alpha H) - \sinh\,(\alpha H)/\alpha]\Big)/K_2(\alpha).
$$

The Fourier transformation yields

$$
\phi(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha x}\,Z(\alpha,y)[F_-(\alpha) + \mathrm{e}^{i\alpha L}\,F_+(\alpha)]d\alpha}{K_2(\alpha)} + \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)}\sinh\,(\alpha(y+H))\,d\alpha}{\alpha}
$$

$$
- \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-i\alpha(x-x_*)}\,Z(\alpha,y)}{K_2(\alpha)} \Big[(\beta\alpha^4 + 1 - d)\cosh\,(\alpha H) - \frac{\sinh\,(\alpha H)}{\alpha}\Big]\,d\alpha.
$$

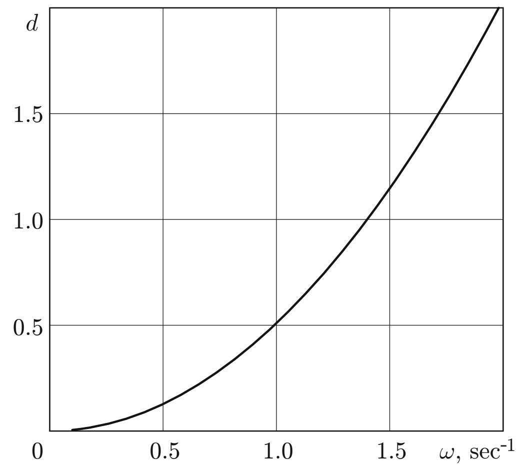

Fig. 1. Dimensionless parameter d versus frequency ω.

The plate deflection $w(x)$ and the dimensionless bending moments $M(x)$ are defined by the formulas

$$w(x) = \sum_{j=-2}^{\infty} \frac{W_j(x)}{K_2'(\alpha_j)}, \qquad M(x) = \frac{\beta l |w''(x)|}{Ld},$$

$$W_j(x) = -i\, \frac{\mathrm{e}^{i\alpha_j|x-x_*|}}{\cosh\,(\alpha_j H)} + \frac{\alpha_j^2 K_1(\alpha_j) K_+(\alpha_j)}{\beta\alpha_j^4 - d}\,[\eta_j\, \mathrm{e}^{i\alpha_j x} + \xi_j\, \mathrm{e}^{i\alpha_j (L-x)}].$$

The first term in $W_j(x)$ is the wave from the vibration source. The values of ξ_j define the complex amplitudes of the waves reflected from the right edge of the plate, and η_j define those reflected from the left edge.

Expressions (2.12) and (2.17) gives the amplitudes of the free-boundary elevation ζ_1 for $x \to -\infty$ and ζ_2 for $x \to \infty$:

$$\zeta_1 = -\frac{i\,\mathrm{e}^{i\gamma x_*}}{\cosh\,(\gamma H) K_1'(\gamma)} - \frac{1}{K_1'(\gamma) K_+(\gamma)}\Big[b_1 + b_2\gamma + N_+(\gamma) - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j L}\, K_+^2(\alpha_j) K_1(\alpha_j) \alpha_j^2 \xi_j}{K_2'(\alpha_j)(\gamma - \alpha_j)}\Big],$$

$$\zeta_2 = -\frac{i\,\mathrm{e}^{-i\gamma x_*}}{\cosh\,(\gamma H) K_1'(\gamma)} - \frac{\mathrm{e}^{-i\gamma L}}{K_1'(\gamma) K_+(\gamma)}\Big[a_1 - a_2\gamma + L_-(-\gamma) - \sum_{j=-2}^{\infty} \frac{\mathrm{e}^{i\alpha_j L}\, K_+^2(\alpha_j) K_1(\alpha_j) \alpha_j^2 \eta_j}{K_2'(\alpha_j)(\gamma - \alpha_j)}\Big],$$

$$L_-(-\gamma) = -\frac{i\,\mathrm{e}^{i\gamma(L-x_*)}\, K_+(\gamma)}{\cosh\,(\gamma H)} - L_+(-\gamma), \qquad N_+(\gamma) = -\frac{i\,\mathrm{e}^{i\gamma x_*}\, K_+(\gamma)}{\cosh\,(\gamma H)} - N_-(\gamma).$$

We now consider the general case. Let the bottom have a segment $[x_1, x_2]$ which vibrates periodically and vertically with a specified displacement law $u(x), x \in [x_1, x_2]$. In this case, multiplying the obtained solution by $u(x_*)$ and integrating the result by x_*, we find the solution for the general case. In the case where the vibrating bottom segment is under the plate edge, the sums in the expressions for $L_+(\alpha)$ and $N_-(\alpha)$ converge weakly; therefore, in calculating their values by formulas (2.7) and (2.8), one first needs to integrate over x_* and then to use residue theory.

4. Numerical Results. The calculations were performed for a model airport for the following parameter values: plate rigidity $D = 1.764 \cdot 10^{11}$ N $\cdot$ m^2, length $L_0 = 1000$ m, fluid density $\rho = 1025$ kg/m^3, and plate draft 5 m. The fluid depth was varied. In this case, the dimensionless parameter d is of significance and cannot be omitted (as was done in a number of papers). A curve of the parameter d versus frequency ω is shown in Fig. 1. It is evident that the parameter d increases rapidly with increase in the frequency, and for $\omega > 1$ sec^{-1}, it should be taken into account.

The vertical displacements of the vibrating bottom segment were specified as $u(x) = u_0 \cos^2(\pi(x - x_0)/(2s))$, where x_0 is the center and s is the half-width of the segment ($s = 200$ m). The dependence of the vibration

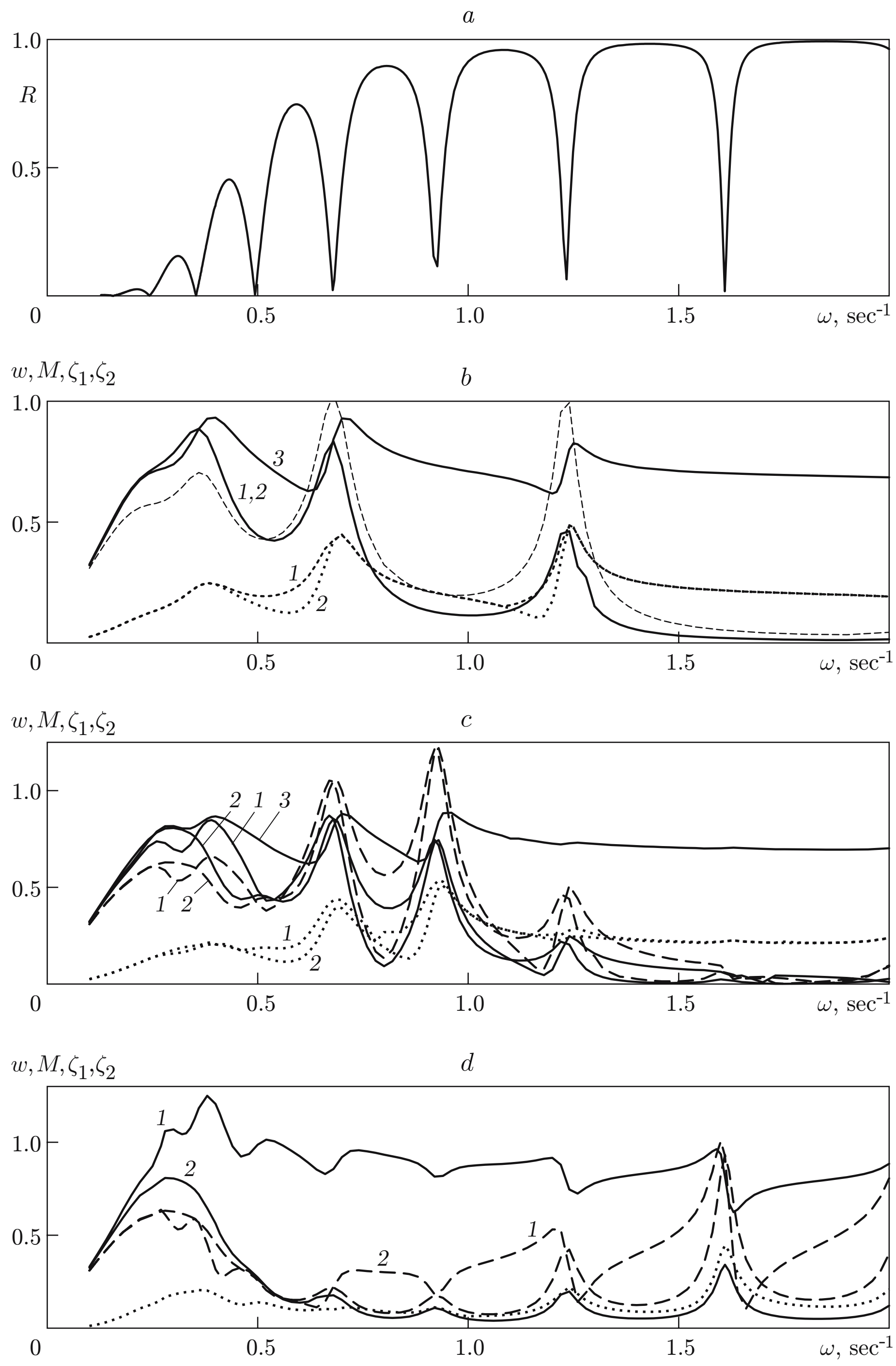

Fig. 2. Reflection coefficient R in the problem of surface-wave diffraction (a), the plate deflection amplitude w, the moment M, and the free-boundary elevation in the far-field region (ζ_1, ζ_2) versus frequency for various positions of the vibrating segment (b) $x_0 = 500$ m, (c) $x_0 = 300$ m, and (d) $x_0 = 0$).

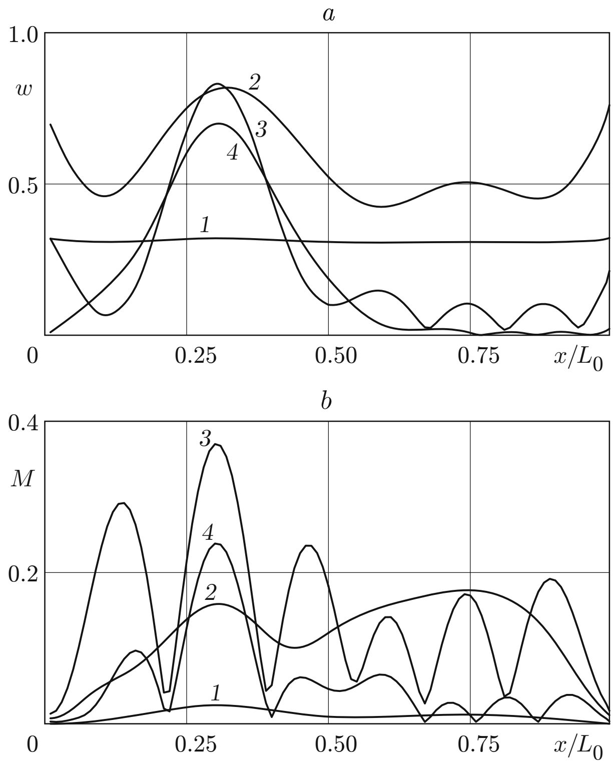

Fig. 3

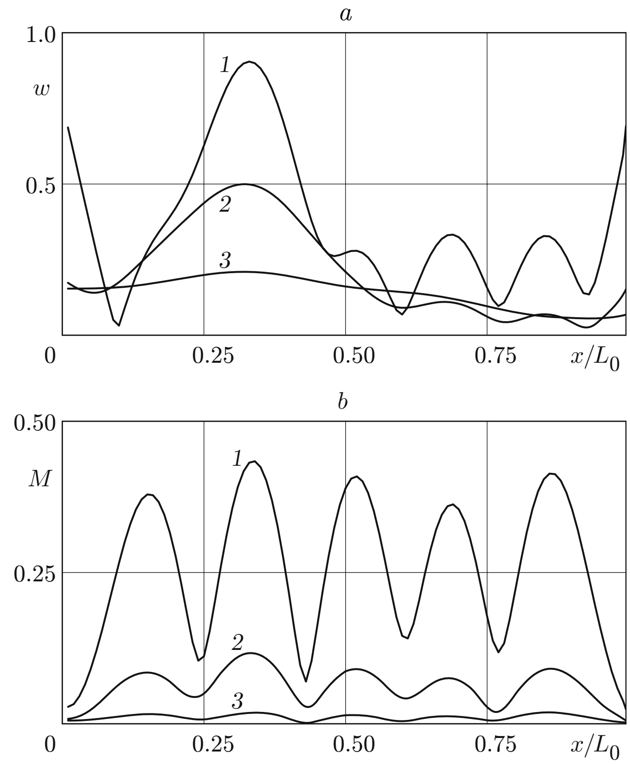

Fig. 4

Fig. 3. Distribution of the plate deflection amplitudes (a) and moment (b) for various frequencies in the asymmetric case: $\omega = 0.1$ (1), 0.3 (2), 1 (3), and 2 sec^{-1} (4).

Fig. 4. Effect of the fluid depth on the plate-deflection amplitude (a) and moment (b) in the asymmetric case for $\omega = 0.7\ \text{sec}^{-1}$: $H_0 = 100$ (1),=200 (2), and 500 m (3).

amplitudes of the fluid and the plate on the frequency, the position of the vibrating segment, and the fluid depth was studied numerically.

Figure 2a gives a curve of the reflection coefficient R versus frequency in the problem of surface-waves diffraction for a fluid of depth $H_0 = 100$ m (the calculations were performed using the technique of [7]). In [8], it is shown that in the case of shallow water, zero values of the reflection coefficient correspond to the diffraction frequencies (resonance frequencies) at which the vibration amplitudes of the plate and the fluids are maximal in the diffraction problem. Obviously, this is also valid for a fluid of finite depth. An increase in the vibration amplitudes of the plate and the fluid at the diffraction frequencies is also noted in the problem of forced vibrations of a floating plate under periodic loading [9].

Figure 2b–d shows curves of the fluid and plate vibration amplitudes versus frequency for $x_0 = 0$ (d) and 500 (b) and 300 m (c) ($s = 200$ m; the fluid depth is $H_0 = 100$ m). The dashed curves 1 and 2 refer to the fluid-elevation amplitudes at infinity on the left and right of the plate, the solid curves 1–3 refer to the plate-deflection amplitudes at the left and right edges and at $x = x_0$; the dotted curves 1 and 2 refer to the maximum amplitude of the dimensionless bending moment on the plate and the amplitude at the point $x = x_0$ (in Fig. 2b, the solid and dashed curves 1 and 2 coincide by virtue of symmetry and in Fig. 2d, $x_0 = 0$, i.e., the left edge is above the center of the vibrating segment). As is evident from Fig. 2, the dependences of the vibration amplitudes of the fluid and plate on frequency are nonmonotonic. The vibration amplitudes increase at the diffraction frequencies (for the symmetric case, the diffraction frequencies corresponding to the asymmetric modes vanish). At the point x_0, the deflection amplitudes vary less significantly than the amplitudes of the edges. The position of the vibrating bottom segment with respect to the plate has a significant effect on the nature of the vibrations.

Figure 3 shows the distribution of the plate- deflection amplitudes and the moment for various frequencies. The fluid depth $H_0 = 100$ m, and $x_0 = 300$ m. At frequencies close to zero, the plate-deflection amplitudes are almost equal at all points (curve 1). With increase in the frequency, the deflection amplitudes become much higher at the point x_0 (the center of the vibrating segment) and at the edges than in the remaining part of the plate (curve 2). With further increase in the frequency, the amplitude of the waves reflected from the edges decrease and the number of ridges and valleys in the middle part of the plate (curves 3 and 4) increase. At high frequencies, the maximum deflection amplitudes are reached above the vibrating bottom segment and the maximum stresses can also be observed on the other segments of the plate.

Figure 4 shows the effect of the fluid depth on the plate-deflection amplitude and the dimensionless bending moment for $\omega = 0.7$ sec^{-1} and $x_0 = 300$ m. As the fluid depth increases, the plate-vibration amplitude decreases.

Thus, the calculation results and a comparison of them with the results for a semi-infinite plate [2] show that the edges have a significant effect on the deflection and stress amplitudes in the plate. For a plate of finite width, the interaction of the propagating modes reflected from the edges and corresponding to the real root α_0 largely determines the deflection amplitudes and stresses in the plate.

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